## PD SUGGESTIONS

> for

MATH TEACHERS

## THE MAIN IDEA's PD Suggestions for Math Teachers

Note that these professional development suggestions are for use with these 3 books:
(1) 5 Principles of the Modern Mathematics Classroom, (2) Making Number Talks Matter, and (3) Mathematical Mindsets


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## I. LEARN the philosophy underlying THREE books for math educators

## A. Teachers Learn the Philosophy in One of the Three Books About Mathematics Education

1. Using a jigsaw approach, divide teachers into three groups and have each group learn about the philosophy in one of the three books below. While it would be ideal to have teachers read the entire book, given time constraints, teachers can focus on the chapters that contain the underlying philosophy of each book indicated below:

- Gerald Aungst's 5 Principles of the Modern Mathematics Classroom (chapters 1-2, pp.1-20)
- Cathy Humphreys' and Ruth Parker's Making Number Talks Matter (chapters 1-3, pp.1-24)
- Jo Boaler's Mathematical Mindsets (chapter 9, pp.171-208)

2. Next, give teachers the chart below with an overview of each author's philosophy. Using the jigsaw approach, have the teachers who read each book introduce the ideas in that book. After all three groups have done this, have the larger group look for what is common among the three (for example, all believe in the importance of having the right mindset about math).

| Humphreys \& Parker | Aungst | Boaler |
| :--- | :--- | :--- |
| 1. Be comfortable with wait time. | 1. Conjecture - Students engage in | 1. Everyone can learn math to the |
| 2. Encourage students to explain concepts (why) not | inquiry, questioning, and problem | highest levels. |
| procedures (how). | finding | 2. Mistakes are valuable. |
| 3. There is no one right approach and mistakes are | 2. Communication - Students read, | 3. Questions are really important. |
| opportunities to learn. | write, speak, and listen when reasoning | 4. Math is about creativity and making |
| 4. Learn to listen. | mathematically | sense. |
| 5. Mathematical understandings will develop over time. | 3. Collaboration - Students work in | 5. Math is about connections and |
| 6. Help students express themselves more clearly. | pairs and groups | communicating. |
| 7. Get students to talk to one another. | 4. Chaos - Class is understandably | 6. Depth is much more important than |
| 9. Confusion and struggle are natural and necessary to | messy when students truly struggle with | speed. |
| learn. | mathematical concepts | 7. Math class is about learning, not |
| 10. Create a culture where all students feel safe to share | 5. Celebration - The focus is on effort | performing. |
| their mathematical ideas. | over achievement |  |

3. Conduct a group brainstorm about how a classroom with the mathematical approaches described above would differ from a more traditional math classroom.

| Traditional mathematics classroom | Classroom that follows the approaches in the 3 books |
| :--- | :--- |
| Ex. The goal is for students to get the right answers and get them quickly. |  |
| Ex. Communication is one way with the teacher explaining a procedure or |  |
| algorithm to the students. |  |
| Ex. Students mostly work and are assessed individually. |  |

## II. DO some of the math problems that reflect these new approaches

The best (and most fun!) way for math teachers to truly learn the ideas in these books is to have them actually do some of the math problems using these new approaches.

## A. Participate in a Number Talk as if the Teachers were the Students (Making Number Talks Matter)

1. If you have someone who already has experience with Number Talks or would like to take the lead, read one of the chapters in Making Number Talks Matter, and act as the teacher, then you can have that person facilitate this section. If not, simply ask teachers to do the following problem, in their heads, without using the traditional algorithm (as a reminder, Number Talks can work with students at all ages as you will see college students discuss the problem below in a video in Step 3!):

[^0]2. Next, have teachers share all of the different methods they used to solve this problem ( $18 \times 5$ ) and have the facilitator record (using pictures if possible) all of the different solutions.
3. Have teachers watch and then discuss this video of Jo Boaler introducing Number Talks, Stanford students solving this problem (18x5), and explaining their thinking. (The video is 15 minutes, but you can share just the first 7 minutes):
https://www.youcubed.org/from-stanford-onlines-how-to-learn-math-for-teachers-and-parents-number-talks/
B. Experience the Difference Between an "Exercise" and a "Problem" (5 Principles of the Modern Mathematics Classroom)

1. Create sets of cards from 0 to 9 or use decks of cards (the Jack can be the 0 ). Then divide teachers into pairs and have them play this game. After playing it several times, have them describe the strategy they used to win:

You and your friends are going to play a game using cards numbered from 0 to 9 . On your turn, draw 3 cards from the facedown deck, one at a time. The object is to make the largest 2 -digit number with your cards, with one card being discarded. The catch is you must decide where to put each digit before drawing the next: tens place, ones place, or discard. If you draw a 4 as your first card, where should you write it, and why?
2. Next have teachers individually solve the following:

Miguel collects baseball cards. Last week he had 217 cards. Today, his aunt gave him two dozen more. How many cards does he have now?
3. As a group, discuss how the two math tasks differ. During the discussion, share that Aungst calls the first one a 'problem,' and the second an 'exercise.' Wikipedia defines an exercise in this way:

An "exercise" is "a routine application of ... mathematics to a stated challenge. Teachers assign mathematical exercises to develop the skills of their students."
4. Ask math teachers to come to future math team (or grade level) meetings with a new problem each time for the teachers to solve. Bring a calendar of all future meetings, sign up teachers, and provide them with these resources to help them find problems:

Bedtime Math, Dan Meyer's Three Act Problems, http://www.math.com/teachers/recreational.html, http://www.insidemathematics.org/problems-of-the-month (problems of the month organized by CCSS), http://math.com/teachers/POW.html, and more.

## C. Have Teachers engage in a Rich Mathematical Task (Mathematical Mindsets)

1. Have teachers solve and discuss one of the problems in Chapter 5 of Boaler's book. A few that only involve pencil and paper are:

- How many ways can you create a rectangle with an area of 24 ?
- Can you make all of the numbers from 1 to 20 using four 4's and any operation. For example:

$$
\sqrt{4}+\sqrt{4}+\frac{4}{4}=5
$$

2. Share Boaler's 6 ways teachers can adapt math tasks to boost conceptual understanding and engagement. Discuss what each of these mean and have teachers share ways they have included any of these design elements in math tasks they've assigned.
1) Open the math task to include multiple pathways (e.g., "You know the rule for $1 \div \frac{2}{3}$. Now make sense of your answer.")
2) Make it an inquiry task (e.g., Instead of find the area of a $12 \times 4$ rectangle, ask how many rectangles you can find with an area of 24.)
3) Ask the problem before teaching the method (e.g., Ask calculus students to find the volume of a lemon before teaching them how to find the area under a curve.)
4) Add a visual component (Have students draw diagrams, pictures, or use objects like multilink cubes and algebra tiles.)
5) Make the floor low and ceiling high (Give a problem everyone can solve but extend it by asking those who finish to create a new question that is similar but more difficult.)
6) Require students to convince and reason (Require that students give more information than just an answer on its own.)

Ask teachers to look at the rectangle or fours problem they solved above and discuss the design features included in this problem. (Note that not all math problems need to incorporate all 6 of these features to be good problems!)

## III. TEACH (or plan to teach) math using a new approach

## A. Have teachers create and role-play conducting a Number Talk

1. First, have teachers choose a topic (subtraction, addition, multiplication, division, or fractions/decimals/percents) to plan a Number Talk for. Then have them read the corresponding chapter in Making Number Talks Matter.
2. Next, have teachers create a 15 -minute Number Talk by choosing a problem (and they can use one from the chapter), anticipating possible strategies students might use for solving the problem, and recording different strategies they think students might use. (Yes, they should actually write these out.)
3. Then have teachers role-play conducting a Number Talk with other math teachers who will act as students. Have one math teacher not participate as a student and instead observe and look for the following in the chart below. Debrief after the role-play, then give everyone a chance to role-play a teacher conducting a Number Talk.
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1. Be comfortable with wait time.
2. Encourage students to explain concepts (why) not procedures (how).
3. There is no one right approach and mistakes are opportunities to learn.
4. Learn to listen.
5. Mathematical understandings will develop over time.
6. Help students express themselves more clearly.
7. Get students to talk to one another.
9. Confusion and struggle are a natural, necessary, and even desirable part of
learning mathematics.
10. Create a learning environment where all students feel safe to share their
mathematical ideas.
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## Observations of these Guidelines in the Role-Play

## B. Have teachers choose a math task they already use with students and increase the level of rigor

1. In 5 Principles of the Modern Mathematics Classroom, Aungst introduces the difference between an 'exercise' and a 'problem.' If we want to include more "problems" - that is, if we want students to develop a deeper understanding of math concepts -- we need to consider the rigor of the problems we assign. One useful tool to use is Norman Webb's Depth of Knowledge. Have teachers look at Webb's descriptions of the different levels of thinking along with the types of math problems that fit each of the first three levels.

| Webb's Depth of Knowledge (DOK) | Example from Area and Perimeter | Example from Quadratics |
| :---: | :---: | :---: |
| Level 1: Recall and Reproduction (recalling basic facts) | Find the perimeter of a rectangle that measures 4 units by 8 units. | Find the roots of the equation: $y=3(x-4)^{2}-3$ |
| Level 2: Skills and Concepts (involves some decisions and skills such as comparing, organizing, and estimating) | List the measurements of 3 different rectangles that each has a perimeter of 20 units. | Create 3 equations for quadratics in vertex form which have roots 3 and 5, but have different max or min values. |
| Level 3: Strategic Thinking (involves planning, evidence and more abstract thinking - such as solving a non-routine problem or explaining the reasoning behind a Level 2 problem) | What is the greatest area you can make with a rectangle that has a perimeter of 24 units? | Create a quadratic equation using the template below with the largest maximum value using whole numbers 1 to 9 no more than once each: $\mathrm{Y}=-\square(\mathrm{X}-\square)^{2}+\square$ |

2. Next have teachers choose a math task they currently use with students that fits into Level 1 above. Alone or in pairs, have teachers tweak their tasks to create a Level 2 or even a Level 3 problem they could use with their students.

## C. Have teachers create norms for the first days back to school AND a rich mathematical task

1. Have teachers read Chapter 9 in Mathematical Mindsets. Discuss the chapter, then tweak the 7 norms introduced in this chapter (and outlined on the first page of this PD section (1. Everyone can learn math to the highest levels, etc.)) to fit each teacher's style. Have teachers think through how they want to introduce these during the first days of school. (Note that Boaler has 5 short mindset videos you can use with younger students at youcubed.org.)
2. Have teachers work together to take one of their math tasks and adjust it based on Boaler's 6 design principles in Chapter 5 .

## IV. ASSESS - What to Look For in Math Classrooms

Have your leadership team look at the chart with the overview of the three authors' philosophies at the beginning of this PD outline. Choose five areas your math teachers most need to strengthen. Then create a shared list of LOOK-FORS to use when instructional leaders observe math teachers. Brainstorm a list of LOOK-FORS that align with the five areas your leadership team chose. Below are a few suggestions:

| 5 Philosophical approaches we need to strengthen | What to look for when observing math instruction |
| :--- | :--- |
| Ex. There is no one right approach and mistakes are <br> opportunities to learn. (Humphreys and Parker) | •Teachers explicitly tell students their brains grow when they make mistakes. <br> •Teachers spend time with wrong answers rather than jumping to correct ones. |
| Ex. Chaos - Class is understandably messy when students <br> are truly allowed to struggle with mathematical concepts <br> (Aungst) | • Teachers provide open-ended questions that allow for messiness and failure. <br> Teachers give students plenty of time to experiment, and keep the stakes low <br> Teachers don't immediately rescue students who struggle, but allow for struggle. |
| Ex. Depth is much more important than speed. (Boaler) | •Teachers provide open-ended tasks that emphasize interesting pathways not speed. |


[^0]:    $18 \times 5$

